STUDENT PROJECT

ON

EUCLID'S ELEMENTS

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Euclid's biography

Heath, *History* p. 354: Proclus (410-485, an Athenian philosopher, head of the Platonic school) on Eucl. I, p. 68-20:

> Not much younger than these is Euclid, who put together the *Elements,* collecting many of Eudoxus's theorems, perfecting many of Theaetetus's, and also bringing to irrefragable demonstration the things which were only somewhat loosely proved by his predecessors. This man lived in the time of the first Ptolemy. For Archimedes, who came immediately after the first, makes mention of Euclid; and further they say that Ptolemy once asked him if there was in geometry any shorter way that that of the *Elements,* and he replied that there was no royal road to geometry. He is then younger than the pupils of Plato, but older than Eratosthenes and Archimedes, the latter having been contemporaries, as Eratosthenes somewhere says.
> (Plato died 347 B.C.; Archimedes lived 287-212 B.C.)

Heath, *History* p. 357: Latin author, Stobaeus (5th Century A.D.):

> someone who had begun to read geometry with Euclid, when he had learnt the first theorem, asked Euclid, "what shall I get by learning these things?" Euclid called his slave and said, "Give him threepence, since he must make gain out of what he learns."

Sarton, p. 19: Athenian philosopher, Proclus (410 A.D. - 485): Ptolemy I, king of Egypt, asked Euclid "if there was in geometry any shorter way than that of the *Elements,* and he answered that there was no royal road to geometry."

Heath, *History* p. 355: Arabian author, al-Qifti (d. 1248):

> Euclid, son of Naucrates, and grandson of Zenarchus, called the author of geometry, a philosopher of somewhat ancient date, a Greek by nationality, domiciled at Damascus, born at Tyre, most learned in the science of geometry, published a most excellent and most useful work entitled the foundation or elements of geometry, a subject in which no more general treatise existed before among the Greeks: nay, there was no one even of later date who did not walk in his footsteps and frankly profess his doctrine... For this reason the Greek philosophers used to post up on the doors of their schools the well-known notice, "Let no one come to our school, who has not first learnt the elements of Euclid."

Heath notes that ancient Arabian scholars describe many important Greek scholars as Arabian.

Heath, *History* p. 356: Pappus (end of 3rd Century A.D.): Apollonius [another mathematician] "spent a very long time with the pupils of Euclid at Alexandria"; also: "The four books of Euclid's Conics were completed by Apollonius, who added four more and gave us eight."

Heath, *The Thirteen Books of the Elements*, p. 6: The Arabians pronounced Euclides "Uclides" and thought his name came from two Arabic words: Ucli, which means "key," and Dis, which means "measurement." They thought Euclid's name meant, essentially, "key to geometry."

Euclidean geometry is a mathematical system attributed to the Greek mathematician Euclid of Alexandria. Euclid's text *Elements* is the earliest known systematic discussion of geometry. It has been one of the most influential books in history, as much for its method as for its mathematical content. The method consists of assuming a small set of intuitively appealing axioms, and then proving many other propositions (theorems) from those axioms. Although many of Euclid's results had been stated by earlier Greek mathematicians, Euclid was the first to show how these propositions could be fit together into a comprehensive deductive and logical system.

The *Elements* begin with plane geometry, still taught in secondary school as the first axiomatic system and the first examples of formal proof. The *Elements* goes on to the solid geometry of three dimensions, and Euclidean geometry was subsequently extended to any finite number of dimensions. Much of the *Elements* states results of what is now called number theory, proved using geometrical methods.

For over two thousand years, the adjective "Euclidean" was unnecessary because no other sort of geometry had been conceived. Euclid's axioms seemed so intuitively obvious that any theorem proved from them was deemed true in an absolute sense. Today, however, many other self-consistent non-Euclidean geometries are known, the first ones having been discovered in the early 19th century. It also is no longer taken for granted that Euclidean geometry describes physical space. An implication of Einstein's theory of general relativity is that Euclidean geometry is only a good approximation to the properties of physical space if the gravitational field is not too strong.

Euclid's Elements: Book VI

(idea) by **avjewe** (1.1 hr) (print) **?** 1 **C!** Thu Jun 21 2001 at 22:04:09

Euclid's Elements Book VI : Similar figures and proportions in geometry.

Definitions

Definition 1

> Similar rectilinear figures are such as have their angles severally equal and the sides about the equal angles proportional.

Definition 2

> Two figures are reciprocally related when the sides about corresponding angles

are reciprocally proportional.

Definition 3

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less.

Definition 4

The height of any figure is the perpendicular drawn from the vertex to the base.

Propositions

Proposition 1

Triangles and parallelograms which are under the same height are to one another as their bases.

Proposition 2

If a straight line is drawn parallel to one of the sides of a triangle, then it cuts the sides of the triangle proportionally; and, if the sides of the triangle are cut proportionally, then the line joining the points of section is parallel to the remaining side of the triangle.

Proposition 3

If an angle of a triangle is bisected by a straight line cutting the base, then the segments of the base have the same ratio as the remaining sides of the triangle; and, if segments of the base have the same ratio as the remaining sides of the triangle, then the straight line joining the [vertex to the point of section bisects the angle of the triangle.

Proposition 4

In equiangular triangles the sides about the equal angles are proportional where the corresponding sides are opposite the equal angles.

Proposition 5

If two triangles have their sides proportional, then the triangles are equiangular with the equal angles opposite the corresponding sides.

Proposition 6

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, then the triangles are equiangular and have those angles equal opposite the corresponding sides.

Proposition 7

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, then the triangles are equiangular and have those angles equal the sides about which are proportional.

Proposition 8

If in a right-angled triangle a perpendicular is drawn from the right angle to the base, then the triangles adjoining the perpendicular are similar both to the whole and to one another.

Corollary

If in a right-angled triangle a perpendicular is drawn from the right angle to the base, then the straight line so drawn is a mean proportional between the segments of the base.

Proposition 9

To cut off a prescribed part from a given straight line.

Proposition 10

To cut a given uncut straight line similarly to a given cut straight line.

Proposition 11

To find a third proportional to two given straight lines.

Proposition 12

To find a fourth proportional to three given straight lines.

Proposition 13

To find a mean proportional to two given straight lines.

Proposition 14

In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional; and equiangular parallelograms in which the sides about the equal angles are reciprocally proportional are equal.

Proposition 15

In equal triangles which have one angle equal to one angle the sides about the equal angles are reciprocally proportional; and those triangles which have one angle equal to one angle, and in which the sides about the equal angles are reciprocally proportional, are equal.

Proposition 16

If four straight lines are proportional, then the rectangle contained by the extremes equals the rectangle contained by the means; and, if the rectangle contained by the extremes equals the rectangle contained by the means, then the four straight lines are proportional.

Proposition 17

If three straight lines are proportional, then the rectangle contained by the extremes equals the square on the mean; and, if the rectangle contained by the extremes equals the square on the mean, then the three straight lines are proportional.

Proposition 18

To describe a rectilinear figure similar and similarly situated to a given rectilinear figure on a given straight line.

Proposition 19

Similar triangles are to one another in the duplicate ratio of the corresponding sides.

Corollary

If three straight lines are proportional, then the first is to the third as the figure described on the first is to that which is similar and similarly described on the second.

Proposition 20

Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side.

Corollary Similar rectilinear figures are to one another in the duplicate ratio of the corresponding sides.

Proposition 21

Figures which are similar to the same rectilinear figure are also similar to one another.

Proposition 22

If four straight lines are proportional, then the rectilinear figures similar and similarly described upon them are also proportional; and, if the rectilinear figures similar and similarly described upon them are proportional, then the straight lines are themselves also proportional.

Proposition 23

Equiangular parallelograms have to one another the ratio compounded of the ratios of their sides.

Proposition 24

In any parallelogram the parallelograms about the diameter are similar both to the whole and to one another.

Proposition 25

To construct a figure similar to one given rectilinear figure and equal to another.

Proposition 26

If from a parallelogram there is taken away a parallelogram similar and similarly situated to the whole and having a common angle with it, then it is about the same diameter with the whole.

Proposition 27

Of all the parallelograms applied to the same straight line falling short by parallelogrammic figures similar and similarly situated to that described on the half of the straight line, that parallelogram is greatest which is applied to the half of the straight line and is similar to the difference.

Proposition 28

To apply a parallelogram equal to a given rectilinear figure to a given straight line but falling short by a parallelogram similar to a given one; thus the given rectilinear figure must not be greater than the parallelogram described on the half of the straight line and similar to the given parallelogram.

Proposition 29

To apply a parallelogram equal to a given rectilinear figure to a given straight line but exceeding it by a parallelogram similar to a given one.

Proposition 30

To cut a given finite straight line in extreme and mean ratio.

Proposition 31

In right-angled triangles the figure on the side opposite the right angle equals the sum of the similar and similarly described figures on the sides containing the right angle.

Proposition 32

If two triangles having two sides proportional to two sides are placed together at one angle so that their corresponding sides are also parallel, then the remaining sides of the triangles are in a straight line.

Proposition 33

Angles in equal circles have the same ratio as the circumferences on which they stand whether they stand at the centers or at the circumferences.

Euclid's Elements: Book V <--- Book VI ---> Euclid's Elements: Book VII

Bell, Harold Idris *Egypt from Alexander the Great to the Arab Conquest.* Oxford: Oxford UP, 1948.

Boardman, Griffin, Murray edd. *The Oxford History of the Classical World.* Oxford: Oxford UP, 1986, p. 382.

Dunham, William *Journey Through Genius.* New York: Penguin, 1991.

Gow, James *A Short History of Greek Mathematics.* Cambridge: Cambridge UP, 1884. Reprinted New York: Chelsea Publishing Company, 1968.

Grant, Michael *Ancient History Atlas.* London: Weidenfeld and Nicolson, 1971.

Heath, Sir Thomas L. *Euclid in Greek.* Cambridge: Cambridge UP, 1920.

Heath, Sir Thomas L. *The Thirteen Books of the Elements*. Cambridge: Cambridge UP, 1956.

Heath, Sir Thomas L. *A History of Greek Mathematics*. Oxford: Oxford UP, 1921.

Heiberg, J. L. and Henge, H. edd. *Euclidis Opera Omnia*. Leipzig: Teubner, 1883-1916.

Lindberg, David C. *The Beginnings of Western Science.* Chicago: Chicago UP, 1992

Sarton, George *Ancient Science and Modern Civilization.* New York: Harper, 1954.

Simson, Robert *The elements of Euclid: viz. the first six books, together with the eleventh and twelfth.* Edinburgh: J. Nourse, 1781.

Talbert, Richard J. A. *Atlas of Classical History*. New York: Macmillan, 1985.

Thomas-Stanford, Charles *Early Editions of Euclid's Elements.* London: Oxford UP, 1926.